Uncertainty Quantification Using Evidence Theory in Multidisciplinary Design Optimization

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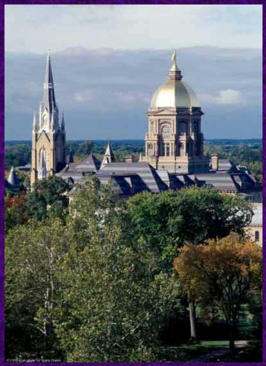

Abstract.

Advances in computational performance have lead to the development of large-scale simulation tools for design. Designs generated using such simulation tools can fail in service if the uncertainty of the simulation tool's performance predictions is not accounted for. In this research an investigation of how uncertainty can be quantified in ***multidisciplinary system analyses*** subject to epistemic uncertainty associated with the disciplinary design tools and input parameters is undertaken. Evidence theory is used to quantify uncertainty. ***In this work, we are introducing multidisciplinary analysis problems as an extension to the challenge problems proposed by Sandia National Laboratories.***

Once uncertainty is characterized mathematically, the designer seeks to optimize the design such that uncertainty is accounted for. The measures of uncertainty given by evidence theory are discretely defined. Performing optimization using traditional gradient-based optimizers is not possible because the sensitivities of the uncertain measures are not properly defined. In this research surrogate models are used to represent the uncertainty measures as continuous functions. A sequential approximate optimization approach is used to drive the optimization process. The methodology is illustrated in application to multidisciplinary example problems.

Motivation

- Uncertainties are introduced in the different phases of the simulation-based design process.
- Increasing attention has focused on differentiating between the different types of uncertainty (i.e., aleatory and epistemic) and on how to model these mathematically.
- For decades, uncertainty in simulation-based design has been formulated solely in terms of probability theory.
- The connection between uncertainty and probability theory is now being questioned as different types of uncertainty have to be accounted for.
- Thus, it is important to use appropriate theories of uncertainty analysis and to investigate their application in simulation-based design environments.

1

Imprecision

Aleatory Uncertainty
(variability, irreducible)

•Inherent variation of the system

•It can be mathematically modeled using probability theory

Epistemic Uncertainty
(reducible)

•Incomplete Information

•Lack of knowledge:

Not enough experimental data

Different mathematical models

Error

2

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Sources of Uncertainty

- There is uncertainty associated with the simulation tools (FEA, CFD, etc) which are used to predict performance states.
- A physical system is analyzed by different fidelity models. Each gives a different result. Hence the uncertainty of the analysis tool. This is known as model form uncertainty.
- Inputs to a simulation tool (parameters) are known to exist in an interval. Due to lack of experimental data it is inappropriate to model the parameter using probability density function. This is known as parametric uncertainty.

Evidence Theory

- Evidence theory is also known as Dempster-Shafer theory.
- It has two complementary uncertainty measures: belief and plausibility.
- Belief is the lower bound whereas Plausibility is the upper bound.
- These measures depend on the available evidence.
- Evidence can be in the form of experimental data, expert opinion, theoretical evidence or consensus among experts regarding the value of a parameter or occurrence of an event.

Axioms and Measures in Evidence Theory

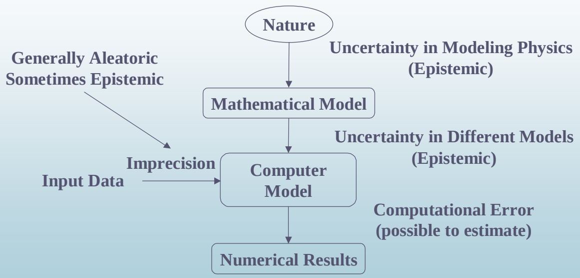
- Let U be the frame of discernment.
- The set of all possible outcomes are $2^U \in U$
- Let m be a function which maps the set 2^U to [0,1] to express basic probability for a set in U.
- The basic probability assignment (BPA) for a set A is represented as m(A).
- Three Axioms of Evidence Theory are:

1. $m(A) \geq 0 \, , A \in 2^U$
2. $m(\varnothing) = 0$
3. $\sum m(A) = 1, \; A \in 2^U$

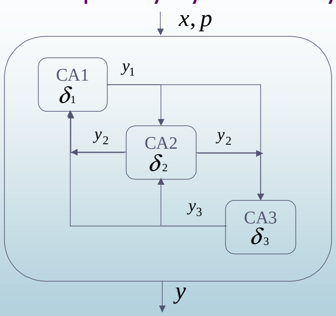
- Belief (Bel) and Plausibility (Pl) are defined as

$$\mathrm{Bel}(A) = \sum m(C) \, , \; C \subset A$$
$$\mathrm{Pl}(A) = \sum m(C) \, , \; C \cap A \neq \varnothing$$

Uncertainty in Design Tools



Multidisciplinary System Analysis



$y = \mathrm{SA}(x)$ requires iterative analysis.

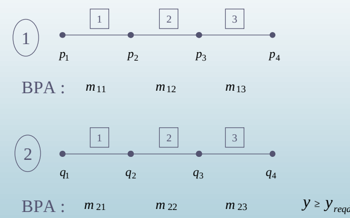
CA's are simulation based design tools.

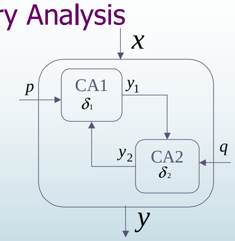
p are uncertain parameters

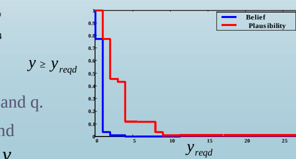
δ_i is tool uncertainty.

Uncertainty Quantification in Multidisciplinary Analysis

Given : Parameters in intervals and corresponding BPA.



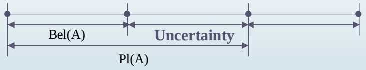




- δ_i intervals and BPA similar to p and q.
- Objective : To obtain the Belief and Plausibility of $y \geq y_{reqd}$

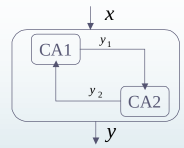
Advantages and Disadvantages

- No assumption is needed on the kind of distribution.
- Accounts for uncertainty by providing two complementary measures.



- Requires system analysis for all possible combination of intervals.
- Belief and Plausibility are discrete uncertainty measures.
- The constraints on these uncertainty measures cannot be directly used in gradient based optimizers.

The Modified Little Problem



Deterministic Optimization :

minimize : $x_1^2 + 10x_2^2 + y_1$

subject to : $y_1 \geq 8$

$y_2 \leq 5$

$-10 \leq x_1 \leq 10$

$0 \leq x_2 \leq 10$

CA1 : $y_1 = x_1^2 + x_2 - 0.2y_2$

CA2 : $y_2 = x_1 - x_2^2 + \sqrt{y_1}$

Optimization with Evidence Theory (OET) :

minimize : $x_1^2 + 10x_2^2 + y_1$

subject to : $Pl(y_1 \geq 8) \geq Pl_{reqd}$

$Pl(y_2 \leq 5) \geq Pl_{reqd}$

$-10 \leq x_1 \leq 10$

$0 \leq x_2 \leq 10$

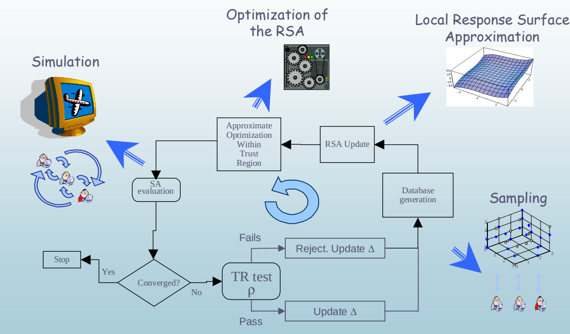
CA1 : $y_1 = x_1^2 + x_2 - 0.2y_2 + \delta_1$

CA2 : $y_2 = x_1 - x_2^2 + \sqrt{y_1} + \delta_2$

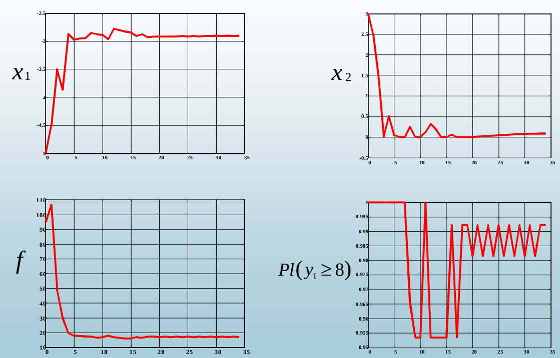
$Pl_{reqd} = 0.99$

- δ_1 and δ_2 are used to account for model form uncertainty
- They are estimated to be in intervals with given BPA.

Sequential Approximate Optimization



Results : The Modified Little Problem



Comparison

	Starting Point	Deterministic Optimization	RBDO $P_{fail} = 0.01$	OET
x_1	-5	-2.82	-3	-2.9
x_2	3	0.05	0.05	0.09
f	94.7	15.98	18.04	17
y_1	29.7	8	9.03	8.5
y_2	-8.5	0.006	0.006	0.007

Closure

- Evidence theory has been used to estimate model form uncertainty and parametric uncertainty in multidisciplinary analysis and optimization.
- Optimization under uncertainty is performed subject to constraints on plausibility in performance.
- Response surface approximations of plausibility facilitates the use of continuous optimization methods.
- A Trust Region Managed sequential approximate optimization algorithm is employed.
- A small multidisciplinary analysis problem illustrates the efficacy of the proposed methodology.
- The methodology has also been implemented in application to a multidisciplinary aircraft concept sizing problem with two uncertain parameters.